Drawing Circles with Rational Quadratic Bezier Curves

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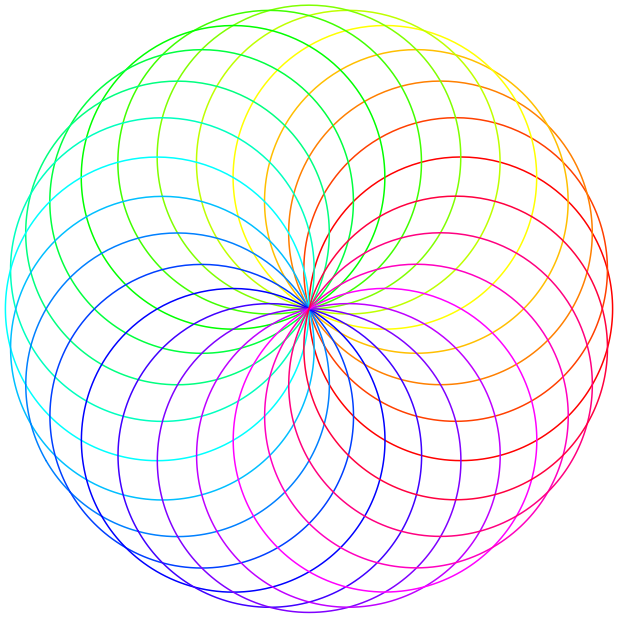
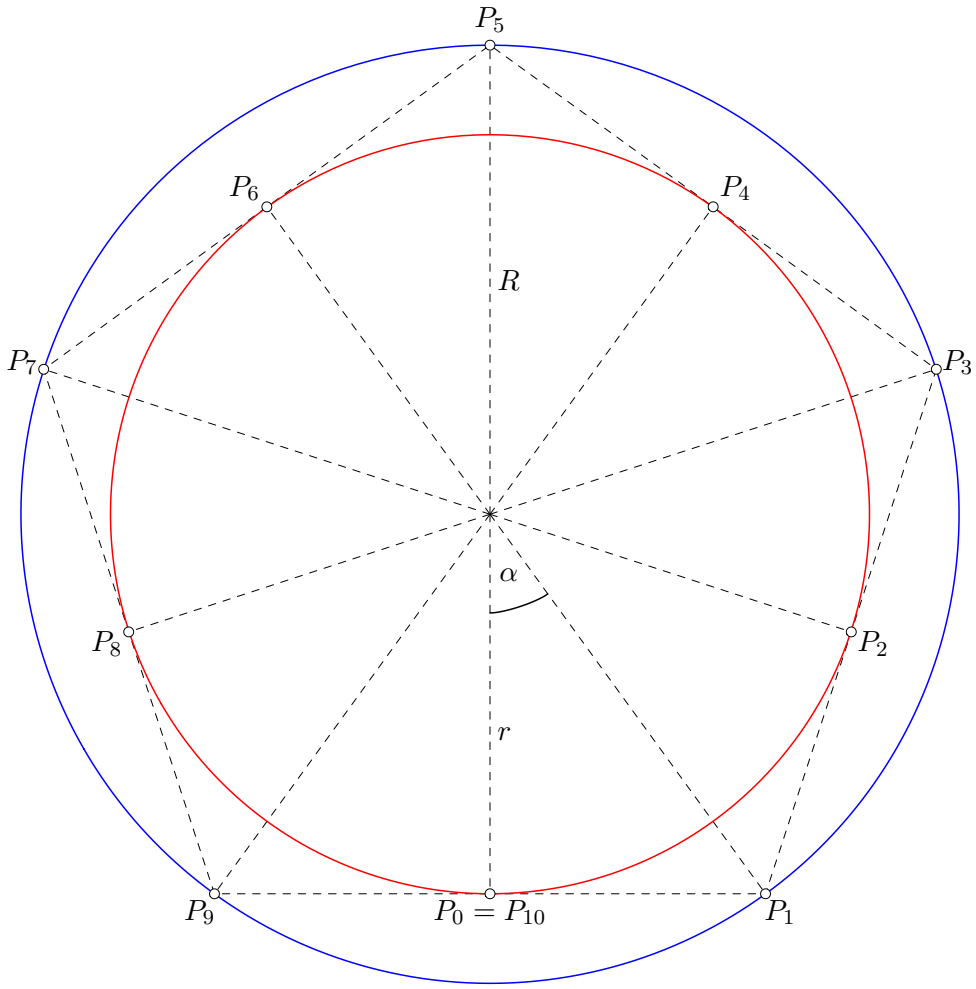


Description

This document explains, how to calculate the bezier points for complete circles. These can be drawn with the `Rcurve` commmand from the `lapdf.sty`. If the weight of the point P_1 is $w = \cos(\alpha)$, where α ist the angle between P_0P_1 and P_1P_2, then the conic will be a circular arc, if also both length P_0P_1 and P_1P_2 are equal.

We have to smothly join several of these arcs together, to get a full circle. Only in the case of two segments, we have have to use one negative weight. In all other cases we only have positive weights. In all of the following calculations and drawings we assume, that the center of the circle lies at the origin.

General calculation scheme

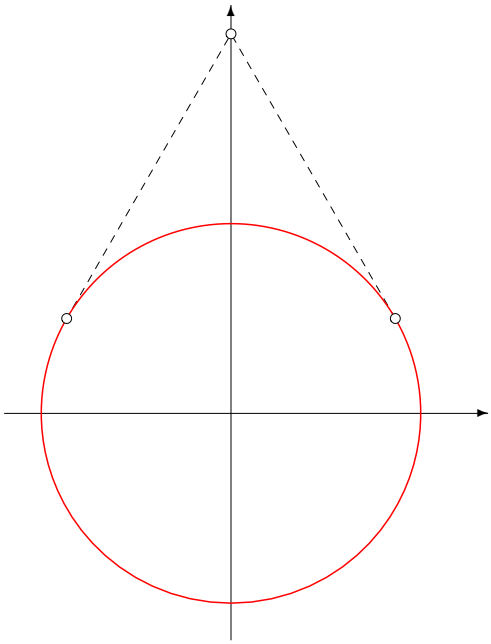


We always put P_0 at the bottom of the circle and all other points follow counterclockwise. This is the general procedure for circle construction with rational quadratic bezier curves (see picture):

1. Set the radius r.
2. Set the number of bezier segments n.
3. Calculate $\alpha = \dfrac{360°}{2n}$.
4. Calculate outer radius $R = \dfrac{r}{\cos(\alpha)}$.
5. Calculate all even bezier points $P_{2i} = \begin{pmatrix} +r \cdot \sin(2i \cdot \alpha) \\ -r \cdot \cos(2i \cdot \alpha) \end{pmatrix}$ for $i = 0 \ldots n$.
6. Calculate odd bezier points $P_{2i+1} = \begin{pmatrix} +R \cdot \sin((2i+1) \cdot \alpha) \\ -R \cdot \cos((2i+1) \cdot \alpha) \end{pmatrix}$ for $i = 0 \ldots n-1$.

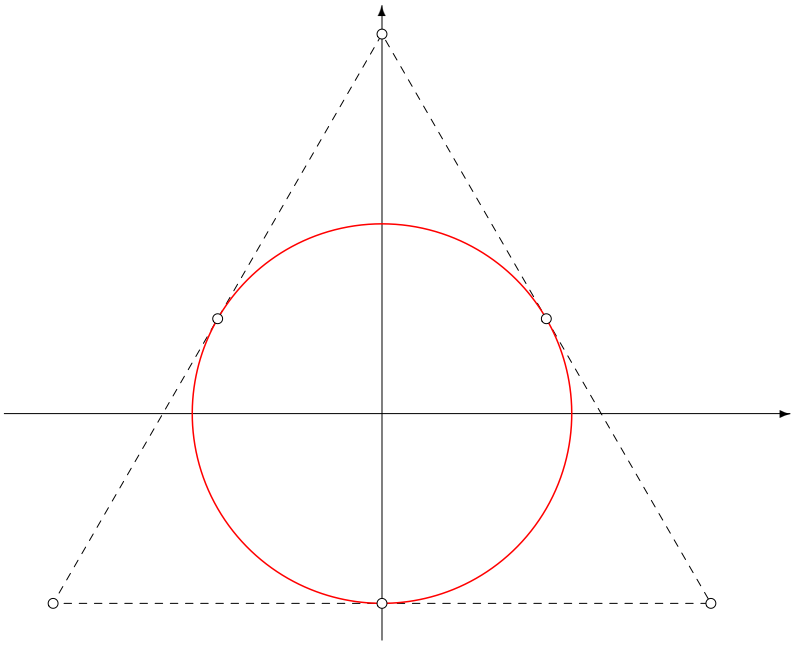
You can control your calculations, if you check your endpoint P_{2n}. This point is equal with P_0. All curves are drawn with the `Rmoveto()` and `Rcurveto()` combination.

2 Segments



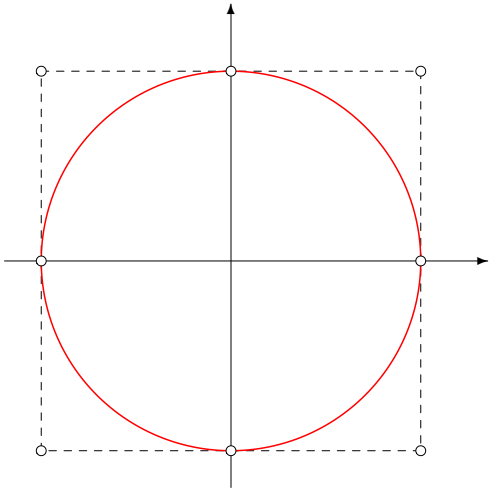
Circle with $2n+1=5$ points ($w_{2n}=1$ and $w_{2n+1}=\pm\cos(60°)=\pm 0.5$).

3 Segments



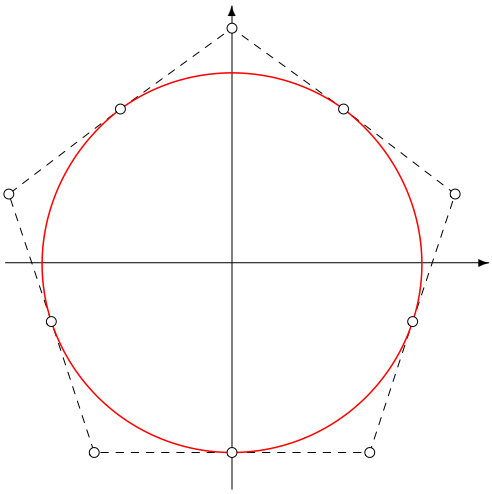
Circle with $2n+1=7$ points ($w_{2n}=1$ and $w_{2n+1}=\cos(60°)=0.5$).

4 Segments



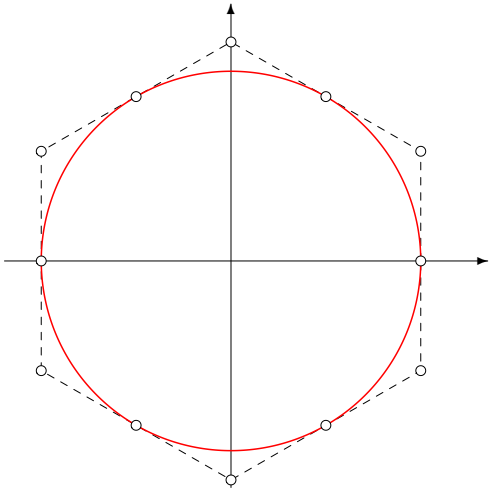
Circle with $2n + 1 = 9$ points ($w_{2n} = 1$ and $w_{2n+1} = \cos(45°) = 0.707$).

5 Segments



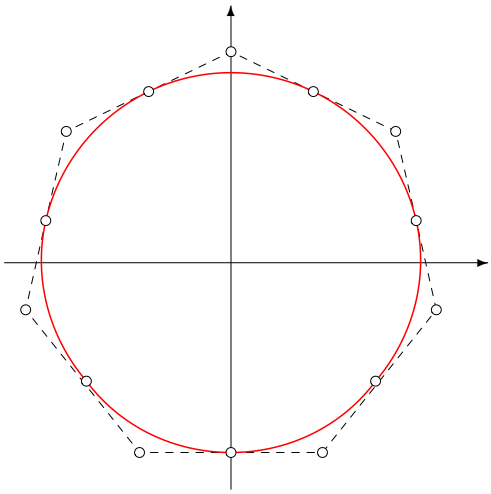
Circle with $2n + 1 = 11$ points ($w_{2n} = 1$ and $w_{2n+1} = \cos(36°) = 0.809$).

6 Segments



Circle with $2n + 1 = 13$ points ($w_{2n} = 1$ and $w_{2n+1} = \cos(30°) = 0.866$).

7 Segments



Circle with $2n + 1 = 15$ points ($w_{2n} = 1$ and $w_{2n+1} = \cos(25.71°) = 0.901$).