# Package 'optical'

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Type Package

Title Optimal Item Calibration

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**Depends** R (>= 4.1.0)

**Description** The restricted optimal design method is implemented to optimally allocate a set of items that require calibration to a group of examinees. The optimization process is based on the method described in detail by Ul Hassan and Miller in their works published in (2019) <doi:10.1177/0146621618824854> and (2021) <doi:10.1016/j.csda.2021.107177>. To use the method, preliminary item characteristics must be provided as input. These characteristics can either be expert guesses or based on previous calibration with a small number of examinees. The item characteristics should be described in the form of parameters for an Item Response Theory (IRT) model. These models can include the Rasch model, the 2-parameter logistic model, the 3-parameter logistic model, or a mixture of these models. The output consists of a set of rules for each item that determine which examinees should be assigned to each item. The efficiency or gain achieved through the optimal design is quantified by comparing it to a random allocation. This comparison allows for an assessment of how much improvement or advantage is gained by using the optimal design approach. This work was supported by the Swedish Research Council (Vetenskapsrådet) Grant 2019-02706.

Author Mahmood Ul Hassan [aut, cre] (<https://orcid.org/0000-0003-2889-0263>), Frank Miller [aut] (<https://orcid.org/0000-0003-4161-7851>)

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**Repository** CRAN

Maintainer Mahmood Ul Hassan <scenic555@gmail.com>

BugReports https://github.com/scenic555/optical/issues

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## **R** topics documented:

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convergenceplot Convergence plot

#### Description

Convergence plots displaying efficiency of design, violations of equivalence theorem, and alpha(Step length used) vs. iteration number. These plot are suitable for monitoring the convergence of optimal item calibration algorithm.

#### Usage

convergenceplot(yyy, refline = c(0.002, 1e-05))

#### Arguments

ууу	a optical object; the output of a call optical()
refline	reference line

#### Details

Convergence plots have three panel.

- First panel monitors efficiency of design vs. iteration number
- · Second panel monitors violations of equivalence theorem vs. iteration number
- · Third panel monitors step size used vs. iteration number

#### Value

A convergence plot is displayed.

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#### drawdesign

#### See Also

drawdesign

#### Examples

```
\# 2PL-models for two items; parameters (a, b)=(1.6, -1) and (1.6, 1), respectively ip <- cbind(c(1.6, 1.6),c(-1, 1))
```

yyy <- optical(ip)</pre>

convergenceplot(yyy, refline=c(0.002, 0.001\*0.005/0.45))

drawdesign Graph for (optimal) design

#### Description

Generate plot for optimal design with four possible layouts. All layouts have design first incl. efficiency vs. random design; then line with item having minimal directional derivative

#### Usage

```
drawdesign(
   yyy,
   ip,
   ablim = 7,
   ylowl = -99999999,
   refline = 0.002,
   textout = TRUE,
   itemnum = NA,
   layout = 1,
   colvec = 1:12
)
```

#### Arguments

ууу	a optical object; the output of a call optical()
ip	matrix with item parameters for all items (number of rows determines number of items; number of column 2 (2PL) or 3 (3PL or mixed 2/3-PL with NA for 2PL-items in third column).
ablim	ability limits; plots will be made in the range [-ablim, ablim]
ylowl	y low level (minimum value of directional derivative shown in the plot)
refline	reference line correspond to desired minimum violation of equivalence theorem
textout	If textout=TRUE (default), the item parameters will be printed if number of items \$<5\$ and the efficiency vs. the random design; if textout=FALSE, no such text is printed

itemnum	number of items
layout	layouts of plots
	• Layout 1: third panel has directional derivatives (cut at ylowl or lowest value of dirdev)
	• Layout 2: third panel has violations of equivalence theorem, should be ide- ally small. Stopping criterion could be <0.002 (refline)
	• Layout 3: third panel monitors efficiency of design vs. iteration number
	• Layout 4: third panel monitors violations of equivalence theorem vs. itera- tion number
	• Layout 5: third panel shows item characteristic curves
	• Layout 0: only one panel with design
colvec	vector of color sequence for items (default is the R-default black, red, green, etc.)

#### Value

An optimal design plot is displayed.

#### See Also

convergenceplot

#### Examples

```
# 2PL-models for two items; parameters (a, b)=(1.6, -1) and (1.6, 1), respectively
ip <- cbind(c(1.6, 1.6),c(-1, 1))</pre>
```

yyy <- optical(ip)</pre>

```
drawdesign(yyy=yyy, ip=ip, ylowl=-1000, refline=0.002, layout=1)
```

efficiency

Efficiency of optimal design

#### Description

This function computes the efficiency of the D, I, and A optimal designs compared to the random design.

#### Usage

```
efficiency(
   yyy,
   ip,
   uncert = FALSE,
   ipop,
```

efficiency

```
oc = "D",
L = NULL,
items = FALSE,
integ = TRUE
```

#### Arguments

)

ууу	a optical object; the output of a call optical()
ip	matrix with item parameters for all items (number of rows determines number of items; number of columns is 2 (for 2PL; or 1PL with common a-parameter when NA in first column from second item) or 3 (for 3PL; or mixed 2/3-PL with NA for 2PL-items in third column)
uncert	if false (default), abilities are assumed to be known; if true, handling of uncer- tainties of Bjermo et al. (2021) is used.
ipop	matrix with item parameters for operational items (used if uncert=TRUE, only).
ос	optimality criterion: "D" (D-optimality, default), "I" (I-optimality with standard normal weight function), "A" (A-optimality).
L	L-matrix (not used for D-optimality)
items	if false (default), only total block efficiency is returned; if true, criteria for opti- mal and random and the efficiency for each item are reported in each column of output. Last column are then total criteria and efficiency. D-, L-, I-, A-optimality
integ	if true (default), integrate() is used for computation of partial information matrices; if false, Riemann rule is used.

#### Value

A numerical value is displayed.

#### See Also

optical

#### Examples

```
\# 2PL-models for two items; parameters (a, b)=(1.6, -1) and (1.6, 1), respectively ip <- cbind(c(1.6, 1.6),c(-1, 1))
```

```
yyy <- optical(ip)</pre>
```

```
# Efficiency of A-optimal design compared to random design
efficiency(yyy, ip, oc="A")
```

```
# Efficiency of D-optimal design compared to random design
efficiency(yyy, ip, oc="D")
```

# Efficiency of I-optimal design compared to random design

```
efficiency(yyy, ip, oc="I")
```

optical

#### Optimal item calibration

#### Description

Calibrate items following a 2PL, 3PL, mixture of 2PL and 3PL model, or 2PL with common discrimination for all items (Rasch-type).

#### Usage

```
optical(
  ip,
  oc = "D",
  uncert = FALSE,
  ipop,
  imf = c(0.005, 0.01, 0.02, 0.05, 0.1, 0.2, 0.45),
 maxiter = rep(300, 6),
 eps = rep(0.002, 6),
  nnn = c(0, 50, 50, 200, 200, 200),
 nsp = c(0.001, 1e-04, 1e-04, 1e-05, 1e-05, 1e-05),
  sss = 0.001,
  falpha = 1.08,
  sdr = TRUE,
  ig = 3,
 ex = 0,
  integ = TRUE,
  show_progress = 1
)
```

#### Arguments

ip	matrix with item parameters for all items (number of rows determines number of items; number of column is 2 (2PL or Rasch-type with NA from second item in first column) or 3 (3PL or mixed 2/3-PL with NA for 2PL-items in third column).
ос	optimality criterion: "D" (D-optimality, default), "I" (I-optimality with standard normal weight function), "A" (A-optimality).
uncert	if false (default), abilities are assumed to be known; if true, handling of uncer- tainties of Bjermo et al. (2021) is used.
ipop	matrix with item parameters for operational items (used if uncert=TRUE, only).
imf	the vector of step-lengths; default c(0.005, 0.01, 0.02, 0.05, 0.1, 0.2, 0.45).
maxiter	maximal number of iterations in each inner loop, the length of this vector defines the number of outer loops.

optical

eps	convergence criterion (maximum violation of eq.th.), vector with value for each iteration in the outer loop, but the same number for all iterations is recommended.
nnn	number of new nodes added at each position in the adaptive grid, vector with value for each iteration in the outer loop (nnn [1] not used).
nsp	node spacing between new nodes, vector with value for each iteration in the outer loop (nsp [1] is the spacing between nodes of the starting grid).
SSS	step size stopping criterion.
falpha	factor alpha for adjusting the step size vector (should be $> 1$ ).
sdr	stop if design repeated (flag TRUE/FALSE).
ig	inner grid between -ig and ig.
ex	intervals of size < ex will be removed (consolidate); if ex=0, no consolidation will be done.
integ	if true (default), integrate() is used for computation of partial information matrices; if false, Riemann rule is used.
show_progress	if 1 (default), no output will be printed for each iteration. If 2, the + symbols will be printed on a line for each Iteration If 3, some output of the function will be printed.

#### Value

Result of this function is a list with following instances:

dd	directional derivatives of optimal solution.
xi	optimal solution.
t	final grid of ability values which was used.
viomax	largest violation of eq.th. from final solution (if < eps, alg. has converged, otherwise not).
h1	interval boundaries for optimal solution.
ht	Refined table of interval boundaries for optimal design with calibrated items and their corresponding probabilities
mooiter	monitoring iterations; information about each iteration to produce convergence plots.
time	running time of algorithm in minutes.
oc	optimality criterion ("D", "I", "A", "L").
L	L-matrix (not for D-optimality).

### Author(s)

Mahmood Ul Hassan (<scenic555@gmail.com>); Frank Miller (<frank.miller@liu.se>)

#### References

Ul Hassan and Miller (2021). An exchange algorithm for optimal calibration of items in computerized achievement tests. *Computational Statistics and Data Analysis*, 157: 107177.

Ul Hassan and Miller (2019). Optimal item calibration for computerized achievement tests. Psychometrika, 84, 1101-1128.

Bjermo, Fackle-Fornius, and Miller (2021). Optimizing Calibration Designs with Uncertainty in Abilities. Manuscript.

#### See Also

drawdesign, convergenceplot, efficiency

#### Examples

```
# 2PL-models for two items; parameters (a, b)=(1.6, -1) and (1.6, 1), respectively
```

```
ip <- cbind(c(1.6, 1.6),c(-1, 1))</pre>
```

yyy <- optical(ip)</pre>

# Table of interval boundaries for D-optimal design with items and # probabilities (expected proportion of examinees in this interval) yyy\$ht

```
# 1PL-models with common discrimination parameter for two items
# (model assumption is that both have same discrimination);
# parameters (a, b)=(1.6, -1) and (1.6, 1), respectively;
# NA for discrimination means that item has same parameter as preceeding item
ip <- cbind(c(1.6, NA), c(-1, 1))</pre>
```

```
yyy <- optical(ip)</pre>
```

# Table of interval boundaries for D-optimal design with items and # probabilities (expected proportion of examinees in this interval) yyy\$ht

```
# 3PL-models for three items; parameters (a, b, c)=(1, 2, 2.5),
# (-1.5, 0.5, 2) and (0.2, 0.1, 0.05), respectively.
ip <- cbind(c(1, 2, 2.5),c(-1.5, 0.5, 2),c(0.2, 0.1, 0.05))</pre>
```

```
yyy <- optical(ip)</pre>
```

# Table of interval boundaries for D-optimal design with items and # probabilities (expected proportion of examinees in this interval) yyy\$ht

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