Package 'PrivateLR'

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Type Package

Title Differentially Private Regularized Logistic Regression Version 1.2-22 Date 2018-03-19 Author Staal A. Vinterbo <Staal.Vinterbo@ntnu.no> Maintainer Staal A. Vinterbo <Staal.Vinterbo@ntnu.no> Description Implements two differentially private algorithms for estimating L2-regularized logistic regression coefficients. A randomized algorithm F is epsilon-differentially private (C. Dwork, Differential Privacy, ICALP 2006 < DOI:10.1007/11681878_14>), if $\log(P(F(D) \text{ in } S)) - \log(P(F(D') \text{ in } S))| \le epsilon$ for any pair D, D' of datasets that differ in exactly one record, any measurable set S, and the randomness is taken over the choices F makes. License GPL (>= 2) NeedsCompilation no **Repository** CRAN Date/Publication 2018-03-20 13:48:35 UTC

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Differentially Private Logistic Regression

Description

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PrivateLR implements two randomized algorithms for estimating L_2 -regularized logistic regression coefficients that allow specifying the maximal effect a single point change in the training data are allowed to have. Specifically, the algorithms take as parameter the maximum allowed change in log-likelihood of producing particular coefficients resulting from any single training data point substitution.

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Usage

```
dplr(object, ...)
## S3 method for class 'formula'
dplr(object, data, lambda=NA, eps=1, verbose=0,
     rp.dim = 0, threshold='fixed', do.scale=FALSE, ...)
## S3 method for class 'numeric'
dplr(object, x, ...)
## S3 method for class 'logical'
dplr(object, x, ...)
## S3 method for class 'factor'
dplr(object, x, ...)
## S3 method for class 'data.frame'
dplr(object, target=ncol(object),...)
## S3 method for class 'matrix'
dplr(object, target=ncol(object),...)
## S3 method for class 'dplr'
predict(object, data, type = "probabilities", ...)
## S3 method for class 'dplr'
summary(object, ...)
## S3 method for class 'dplr'
print.summary(x, ...)
## S3 method for class 'dplr'
print(x, ...)
```

scaled(fml, data)

Arguments

object	can be given as an object of formula, data.frame, matrix, or factor, logical, numeric vector.		
	If a data.frame, matrix is given, this object contains both the dependent vari- able indexed by target as well as the independent variables, of which all are used. If the dependent variable is a factor, the first level is encoded as 0 and all others as 1.		
	In dplr.formula object is an object of class formula or an object that can be coerced into one.		
	If given as a vector, object contains the values of the dependent variable. The vector object can be of class numeric, in which case it must only contain values 0 and 1, logical in which case it is coerced into numeric by as.numeric(object), or be of class factor, in which case it is coerced into numeric by encoding the first factor level as 0 and all the other levels as 1.		
data	a data frame or matrix containing the variables in the model described by formula.		
lambda	the regularization parameter. If NA (default), the smallest regularizer lambda such that $2 \times \log(1 + 1/(4 \times n \times \text{lambda})) == \exp/10$ is used. If eps is 0, then lambda is set to 0.001.		

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eps	the privacy level. The coefficients of the model are computed by a method that guarantees eps-differential privacy. If eps is 0, then non-private regularized logistic regression is performed.	
verbose	regulates how much information is printed, 0 nothing, 1 a little, 2 more.	
rp.dim	if rp.dim is non-zero, random projection is performed on the data before esti- mating the model parameters. If rp.dim is positive, the projection will be onto rp.dim dimensions. If rp.dim is negative, rp.dim is set to $1/2 * (1/2)^{(-2)} * \log(n)$. If rp.dim is larger than the dimensions of the data, a warning is given and no projection is performed.	
threshold	dplr can non-privately estimate the optimal probability threshold for classifica- tion by one of two methods: 'youden', or 'topleft'. The method 'youden' computes the threshold that maximizes the Youden J, while 'topleft' computes the threshold corresponding to the point on the ROC curve that is closest to (0,1). Any other value (default) will result in a threshold of 0.5.	
do.scale	The privacy guarantees are for data where the covariate vectors lie within the unit ball. If do.scale is TRUE, input data will be scaled such that the covariate vectors all lie within the unit ball.	
type	predict can yield two types of results. If type is "probabilities", then prob- abilities are returned, otherwise predictions of class values are returned using the threshold given by the p.tr element of object.	
x	In the print and print.summary, x is an object of class "dplr" or summary.dplr, typically returned by dplr or summary. Otherwise, the parameter x can either be a numeric matrix containing the covariates or dependent variables (one per column) corresponding to the dependent variable object, or a data frame containing a mix of numeric and factor columns. Any factor is internally recoded as contrasts.	
target	the index of the column in data that contains the target values. Default is the last column of data.	
fml	A formula that describes the dimensions of the data that should be scaled into the unit ball.	
	verbose, lambda, and eps parameters. Not used in summary, print, and predict functions. In addition, a Boolean argument op can be given to dplr to select between <i>objective perturbation</i> (op = TRUE, the default) and <i>output perturbation</i> (op = FALSE).	

Details

The function dplr implements logistic regression using the differentially private methods by Chaudhuri, Monteleoni, and Sarwate.

The interface is similar but not identical to that of lm, with the addition of the possibility of supplying a data matrix or data.frame together with a target column index (defaults to ncol(data)).

The returned model instance has a convenience function model\$pred that takes a data matrix or data frame to be classified as input.

The print function currently prints the summary.

The scaled function scales data such that covariate vectors lie within the unit ball. Note that the response variable is put as the last column in the data frame data returned. Also, the response column name might have changed, depending on the left side of the formula given.

Methods details:

A randomized algorithm A, taking a dataset as input, is said to be ϵ -differentially private if it holds that

$$|\log(P(A(D) \in S)) - \log(P(A(D') \in S))| \le \epsilon$$

for any pair of datasets D, D' that differ in exactly one element, and any set S. We now turn to the algorithms implemented by dplr.

Let ||v|| denote the L2 norm of a vector v, and let

$$J(w,\lambda) = ALL(w) + \lambda/2 ||w||^2$$

where ALL(w) is the average logistic loss over the training data of size n and dimension d with labels y and covariates x. L2-regularized logistic regression computes

$$w^* = \arg\min_w J(w, \lambda)$$

for a given λ .

The function dplr implements two approaches to ϵ -differential private L2 regularized logistic regression (see the ... argument op above). The first is *output perturbation*, where we compute

$$w' = w^* + 2/(n\lambda\epsilon)b,$$

where *b* is a *d*-dimensional real vector sampled with probability proportional to $\exp(-\|b\|)$. The second is *objective perturbation*. Let

$$F(w, \lambda, \epsilon) = J(w, \lambda) + 2/(\epsilon n)b^T w$$

where n and b are as above. Let c = 0.25 and let $z = 2\log(1 + c/(\lambda n))$, then if

 $\epsilon - z > 0,$

we compute

$$w' = \arg\min_{w} F(w, \lambda, \epsilon - z)$$

otherwise we compute an adjusted lambda version

$$w' = \arg\min_{w} F(w, c/(n(exp(\epsilon/4) - 1)), \epsilon/2).$$

The logistic regression model coefficients w' are then ϵ -differentially private.

Value

The dplr function returns a class "dplr" list object comprised of elements including:

par the coefficients of the logistic model.

coefficients same as par

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value, counts, convergence, message				
	these are as returned by the optim method.			
CIndex	the area under the ROC curve (aka., C-Index) of the model on its training data.			
eps	the supplied privacy level.			
lambda	the regularization parameter used			
n	the number of data points			
d	the dimensionality of the data points			
pred	a convenience function such that predict(model, data,) is equivalent to model\$pred(data,).			
p.tr	this is the classification probability threshold.			
did.rp	TRUE if random projection was performed.			
rp.dim	if random projection was performed this contains the number of dimensions projected onto. Only present if random projection was performed.			
rp.p	the projection matrix used for random projection. Only present if random pro- jection was performed.			
scaled	TRUE if data was scaled by providing do.scale = TRUE.			
status	a text string indicating the status of the computations. 'ok' means all is well, 'adjusted lambda' means that the regularizer was too small and had to be adjusted, and 'unique.outcomes' means that the response had only one value, resulting in fixed coefficients returned.			

The scaled function returns a list of the following:

data	the scaled data frame
scale	the scaling factor used.

Warning

The privacy level is only guaranteed for the coefficients of the model, not for all the other returned values, and also only in the case when input data points (potentially after expansion of factors) are of L2-norm ≤ 1 . In particular using prediction thresholds estimated using data (methods 'youden' and 'topleft'), as well as built in scaling of data is not guaranteed. Both of these are turned off by default.

Note

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References

Chaudhuri K., Monteleoni C., and Sarwate, A. Differentially Private Empirical Risk Minimization. *JMLR*, 2011, 12, 1069-1109

See Also

glm and predict

Examples

```
data(iris)
```

```
# the following two are equivalent
# and predict Species being any
# but the first factor level.
model <- dplr(iris)</pre>
model <- dplr(Species ~ ., iris)</pre>
# pick a particular factor level and privacy level 2
model <- dplr(I(Species != 'setosa') ~ ., iris, eps=2)</pre>
# The following is again equivalent to the two first
# examples. Note that we need to remove 'Species' from the
# covariate matrix/data frame, and
# that the class reported by summary will now
# not be 'Species' but 'dplr.class'.
model <- dplr(iris$Species, iris[,-5])</pre>
# two equivalent methods to get at the predicted
# probabilities
p <- model$pred(iris)</pre>
p <- predict(model, iris)</pre>
# print a summary of the model. Note that
# only the coefficients are guaranteed
# to be generated in an eps-differentially
# private manner.
```

```
.
summary(model)
```

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